Nature's Dynamic Graphs

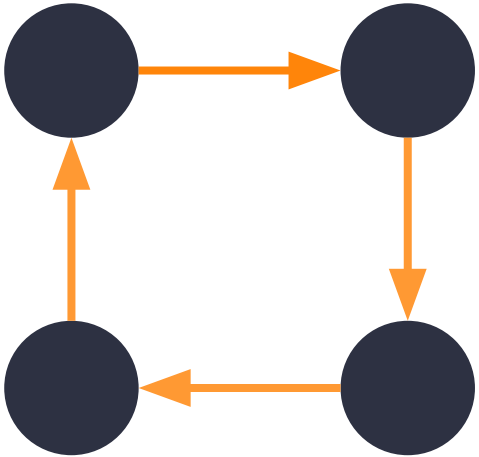
SIAM CSE 2023, February 27 @ 14:30

by Dev Dabke (ddabke@princeton.edu)

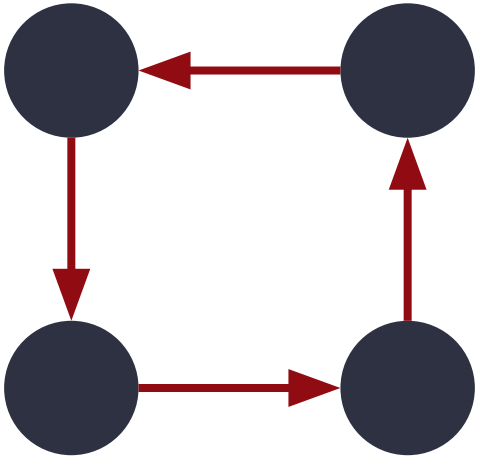
Dynamic Graphs

1. Pose interesting theory questions (because local does not imply global)
2. Arise naturally everywhere
3. Tractable with applied techniques, i.e. ML

I. Local ≠ Global

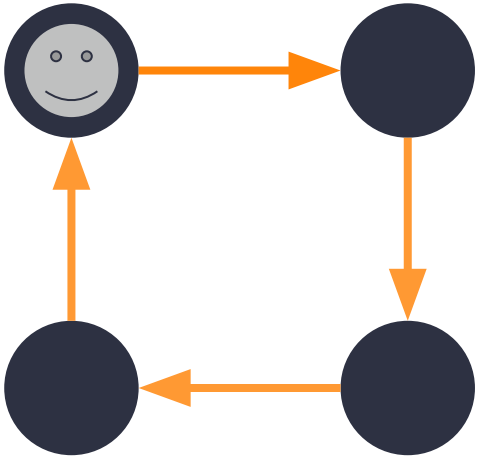


t = 1, 3, 5, ...

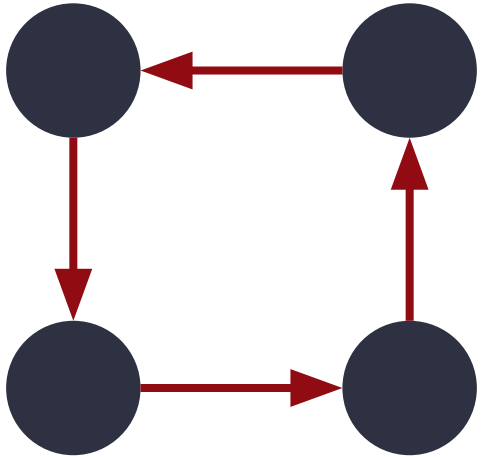


t = 2, 4, 6, ...

The alternating cycle: a discrete-time sequence

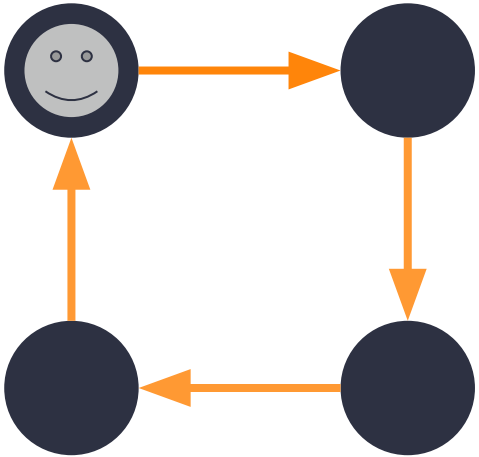


t = 1, 3, 5, ...

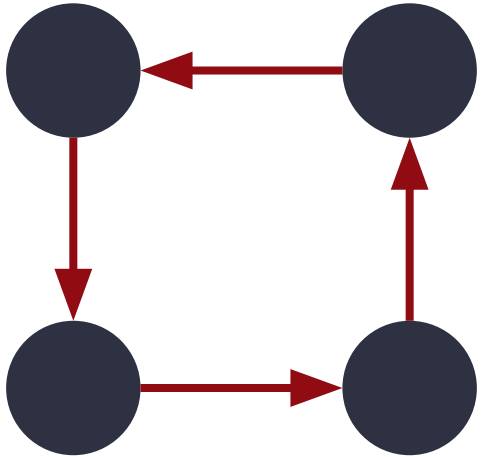


t = 2, 4, 6, ...

Dynamical system: move one edge at each time step

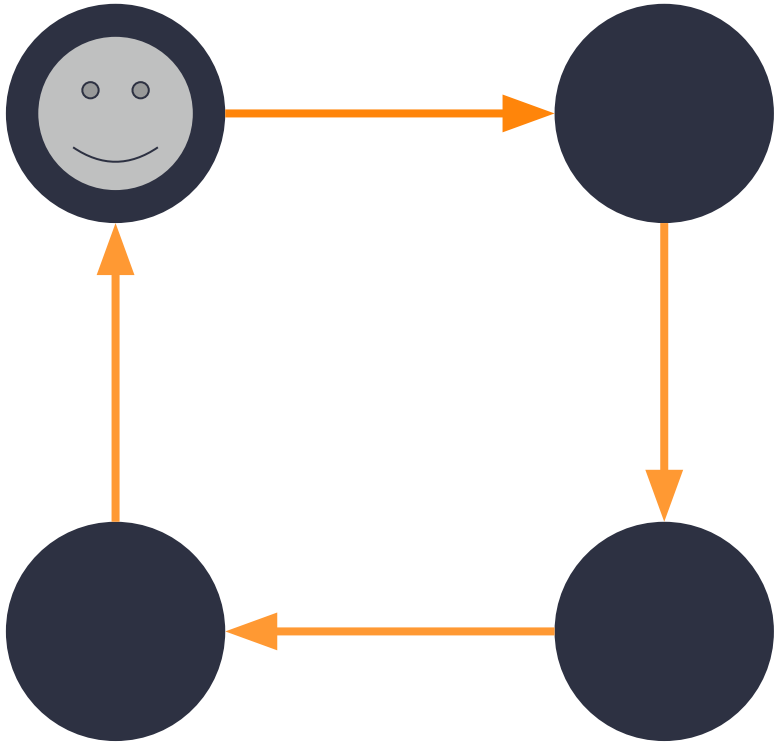


t = 1, 3, 5, ...

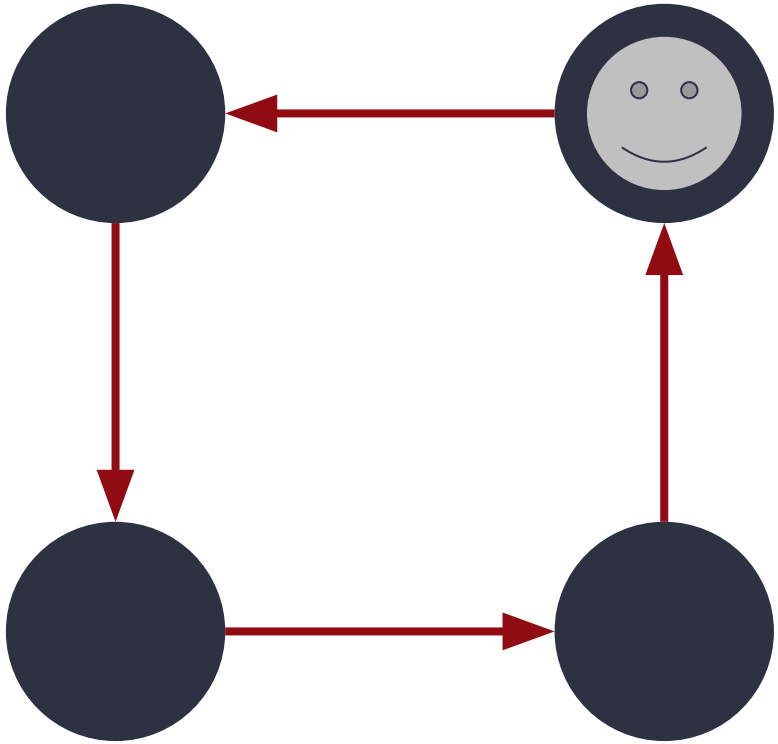


t = 2, 4, 6, ...

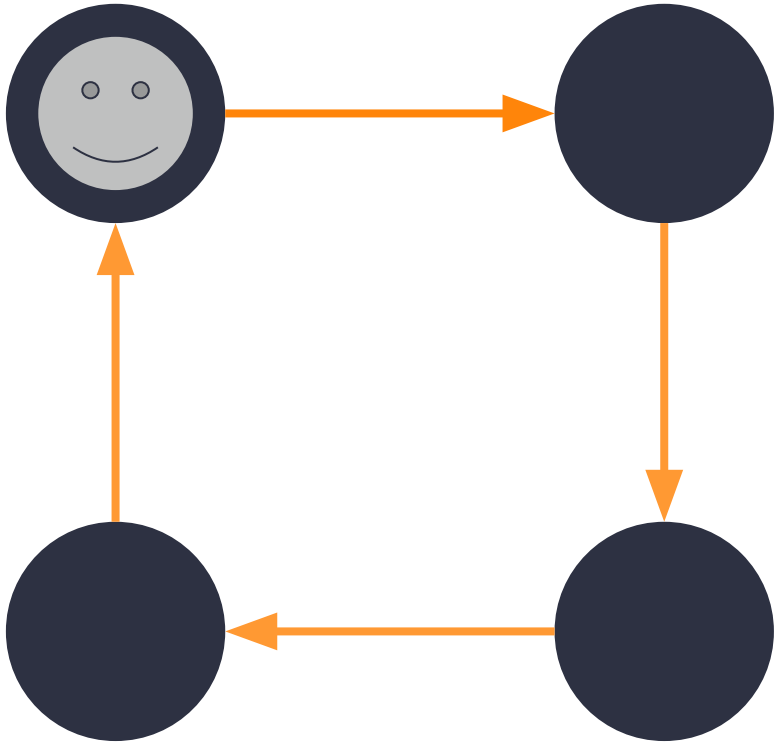
Theorem: max graph diameter is n - 1 => n steps
(for strongly connected graphs)



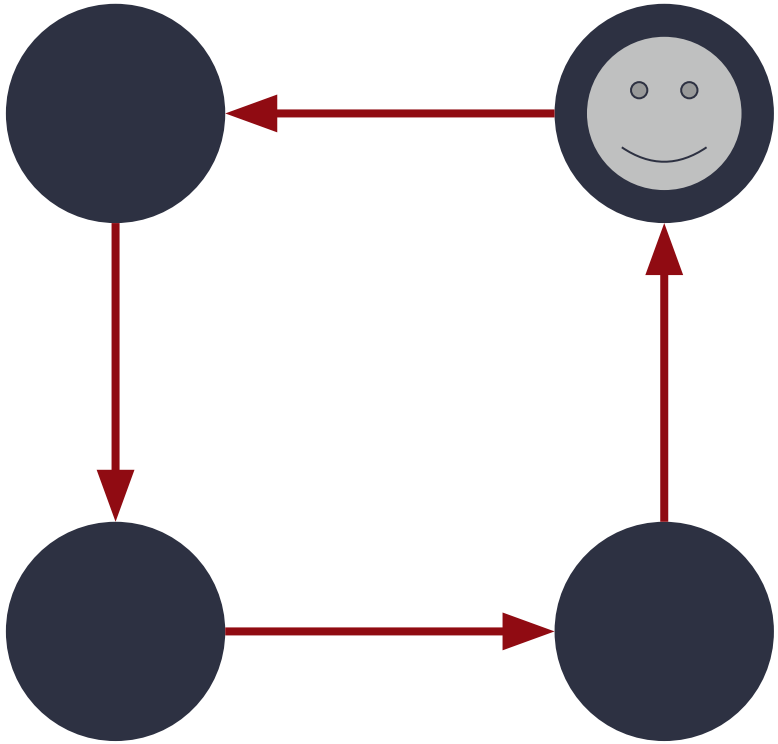
t = 1



t = 2

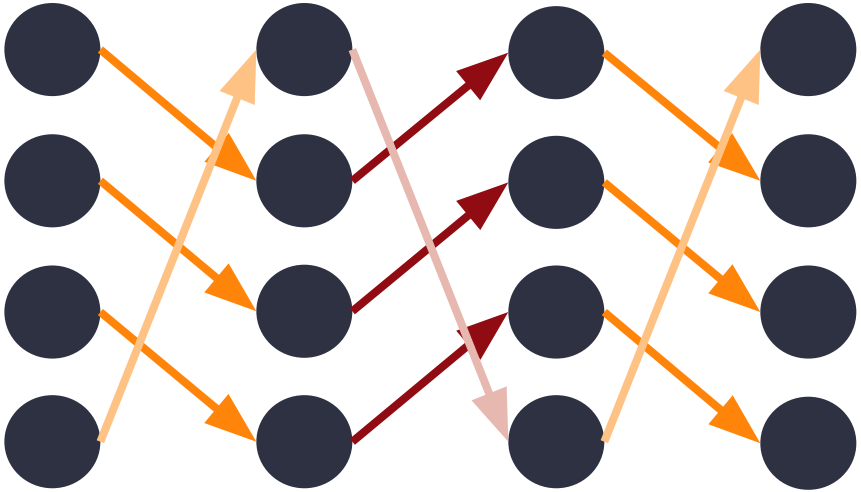


t = 3

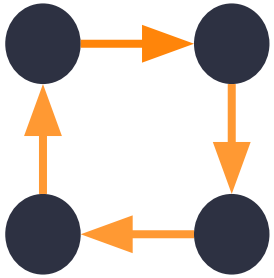


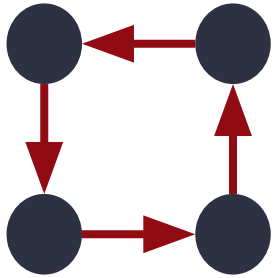
t = 4

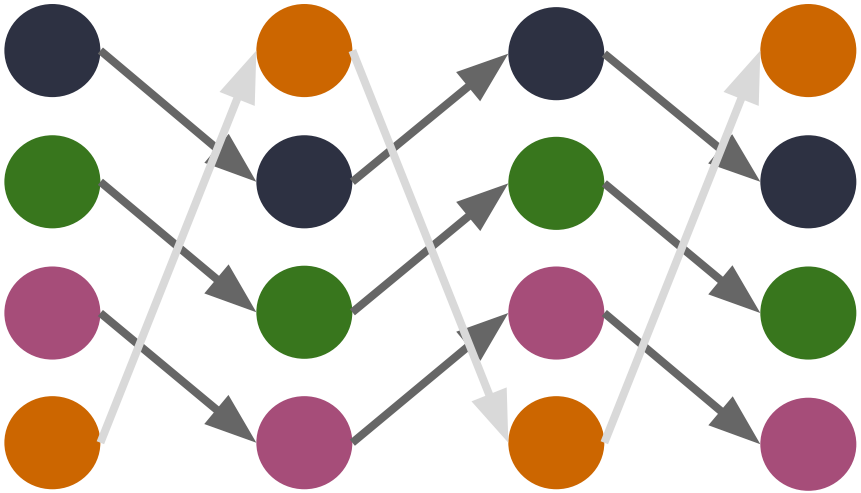
Dynamic diameter is ∞



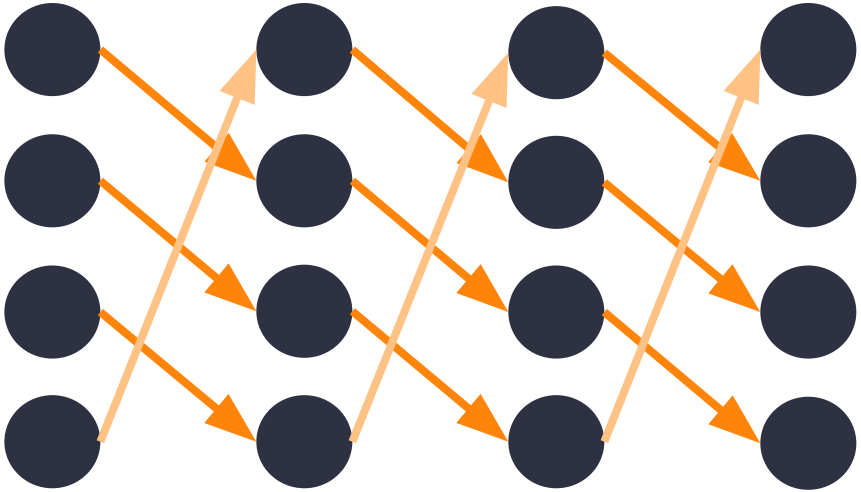
Idea: time-extended graph



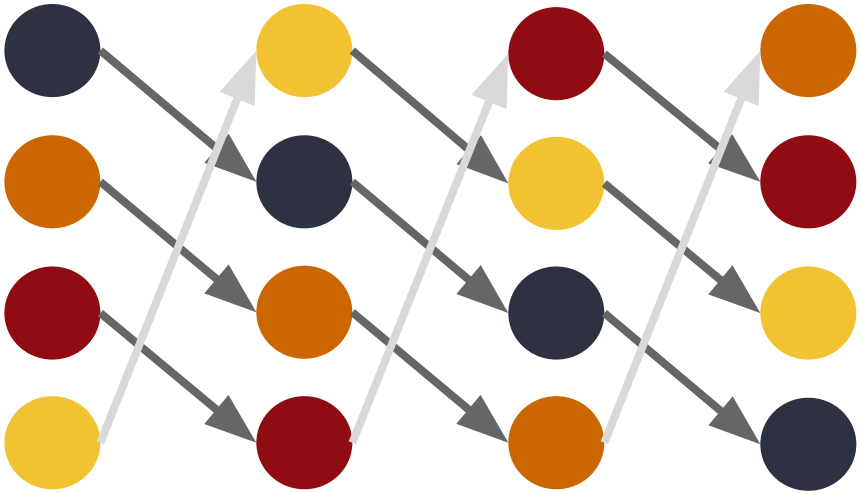




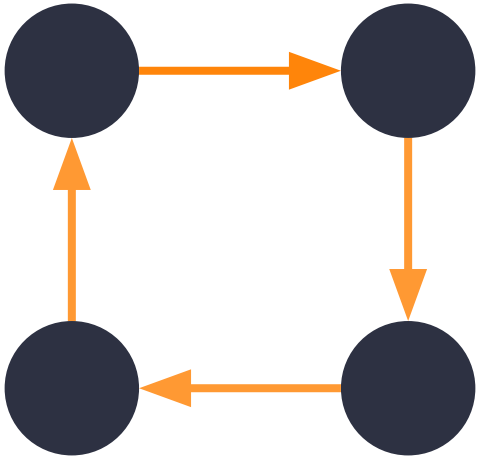
Observation: (weakly) disconnected => dynamically disconnected



Or does it?

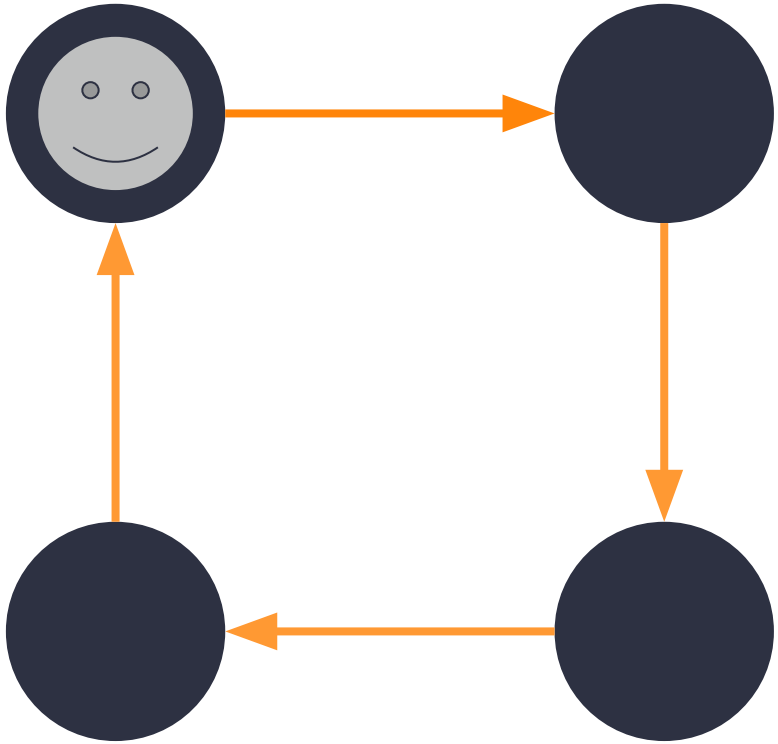


Uh oh ...

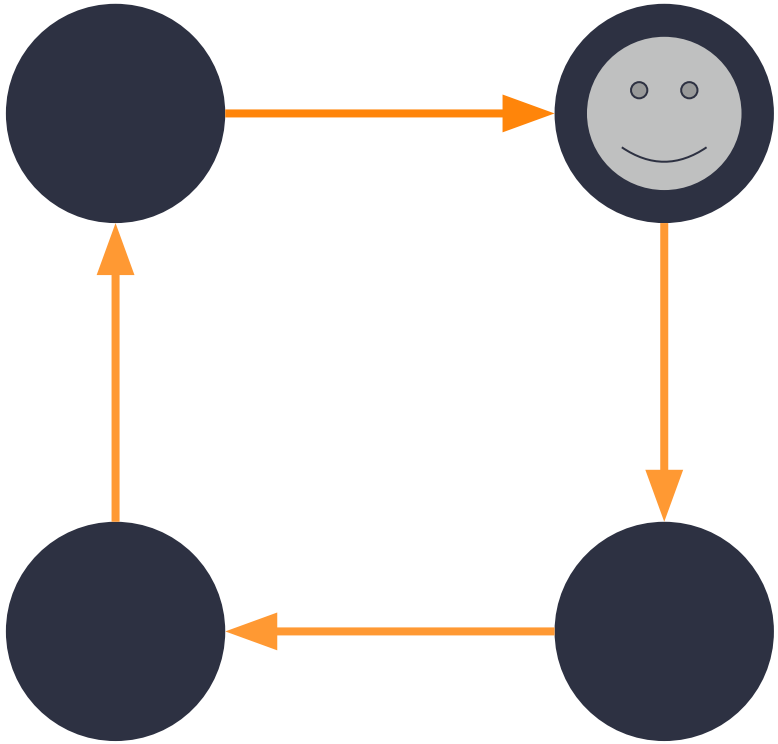


t = 1, 2, 3, 4, ...

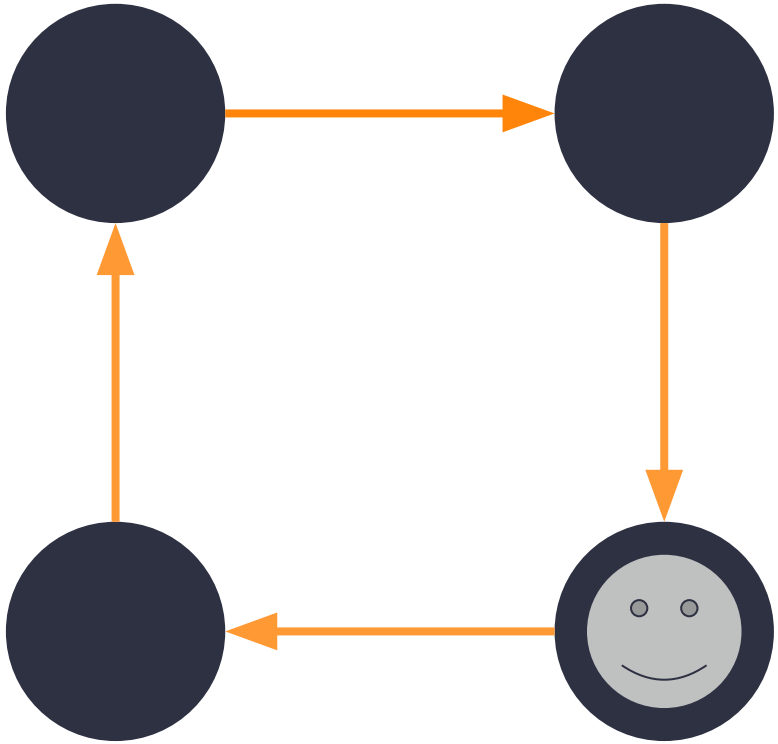
A static cycle



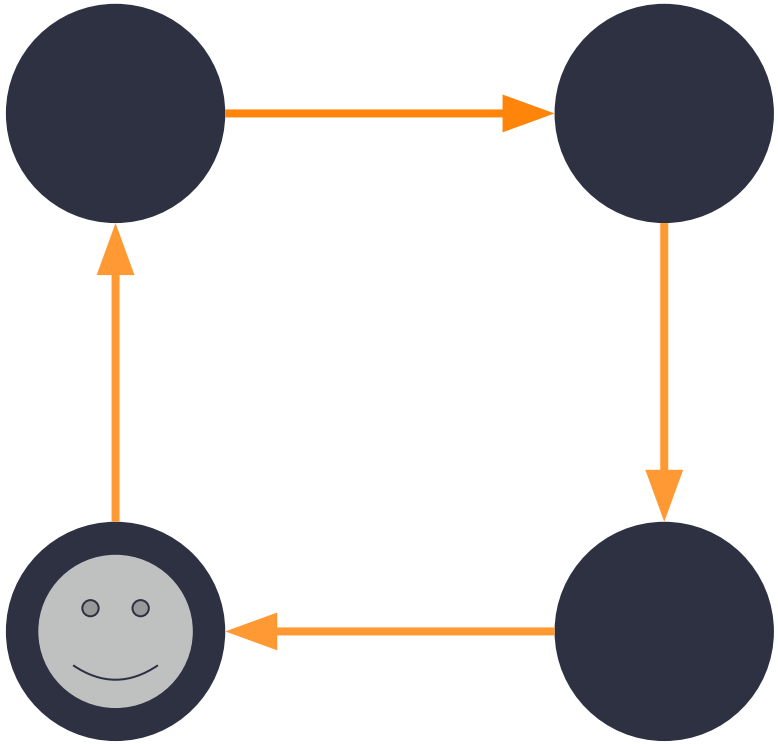
t = 1



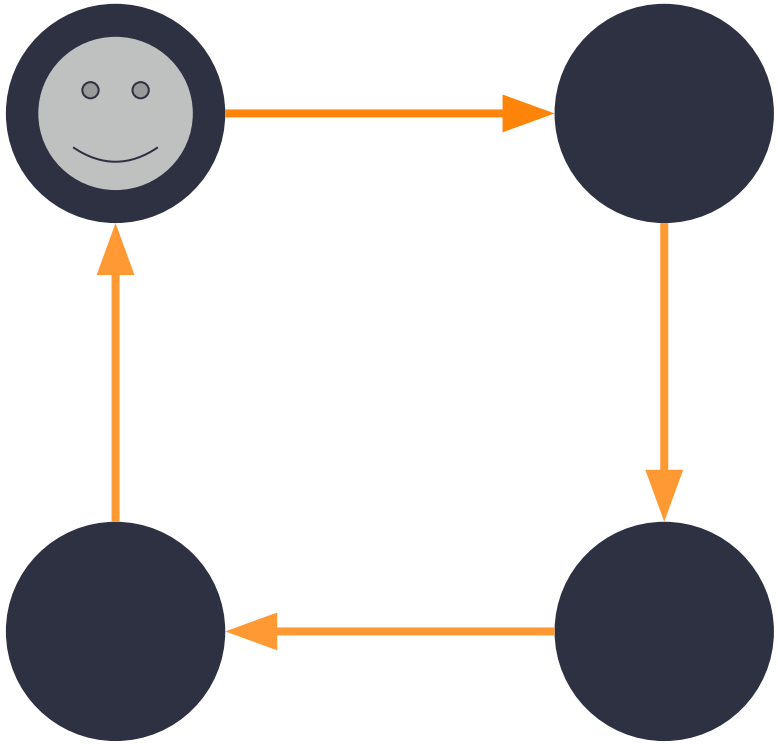
t = 2



t = 3



t = 4



t = 5

$$\mathbf{v} = n * t$$

But, Dijkstra's Still Works

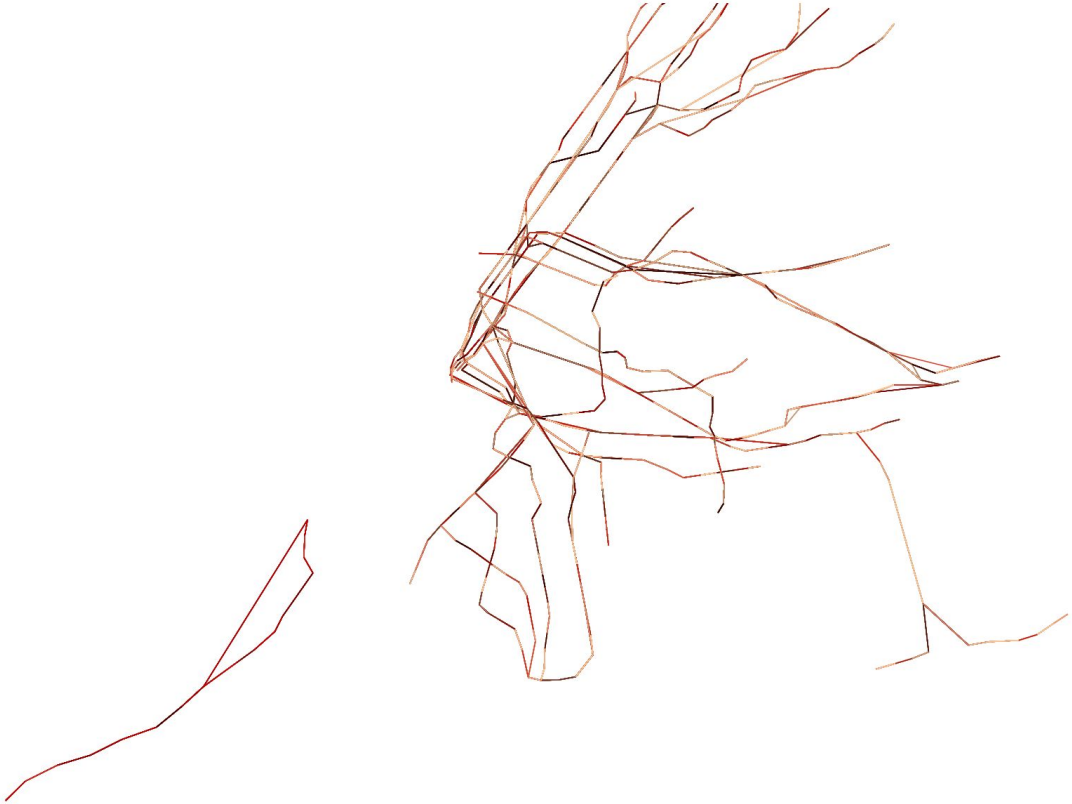
between two edges u, v, we have a function

$f_{uv}(s) : R \rightarrow R$

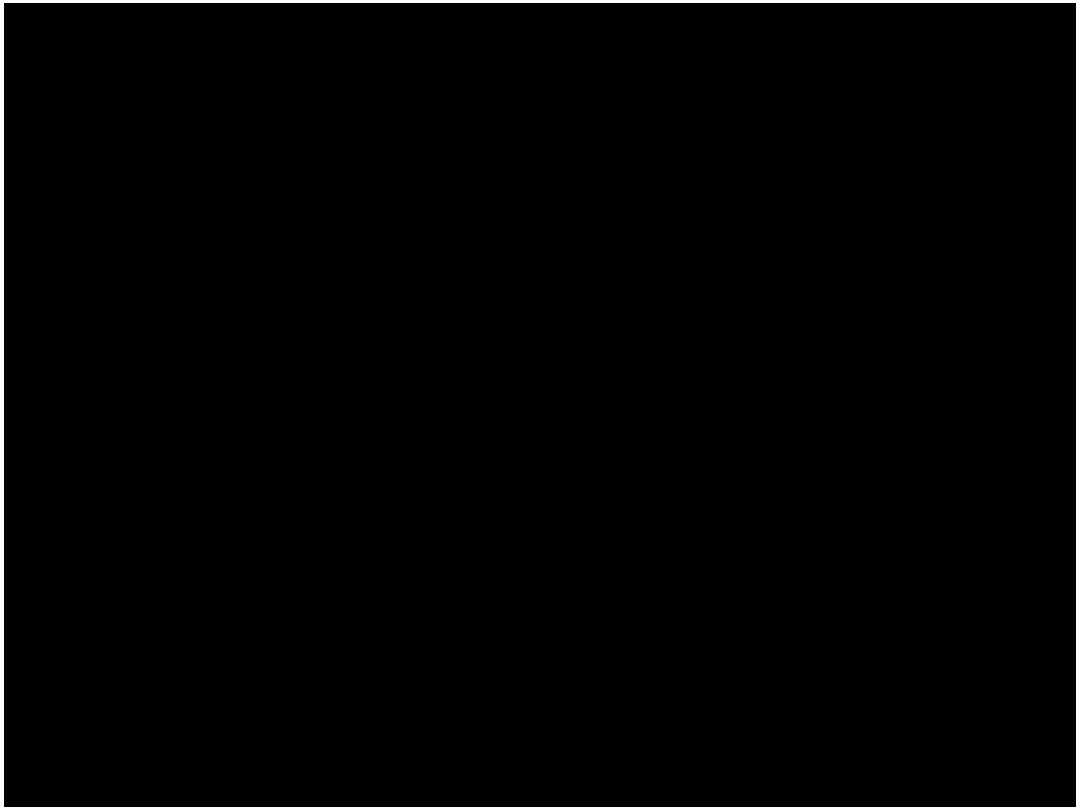
where $f_{uv}(s)$ tells us that given we start traversal at time s, we will arrive by time $f_{uv}(s)$.

condition: $s \leq f_{uv}(s)$

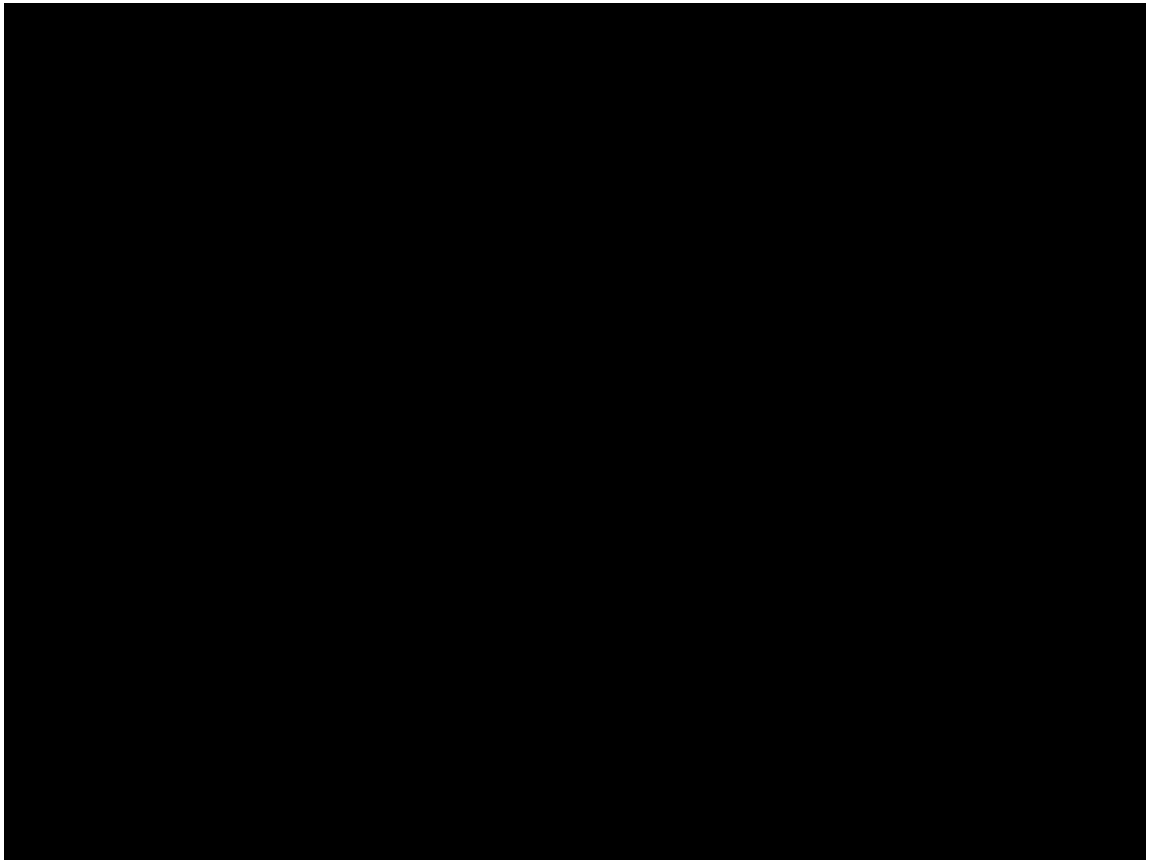
II. Natural Algorithms



Transit Networks (GTFS)

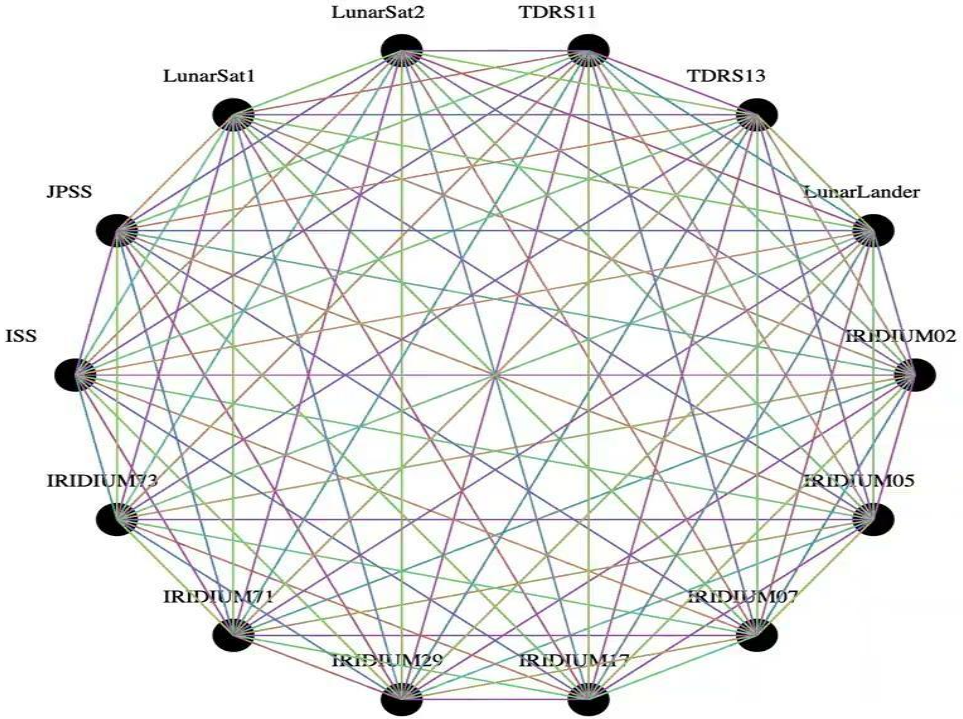


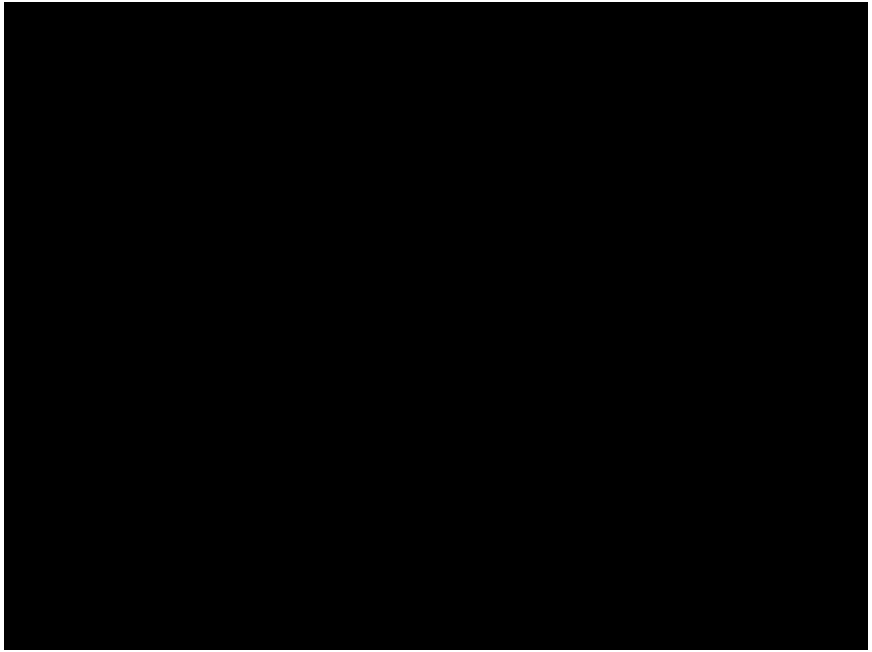
Embryos: spatial and chemical connectivity



Basketball

NASA: Satellites

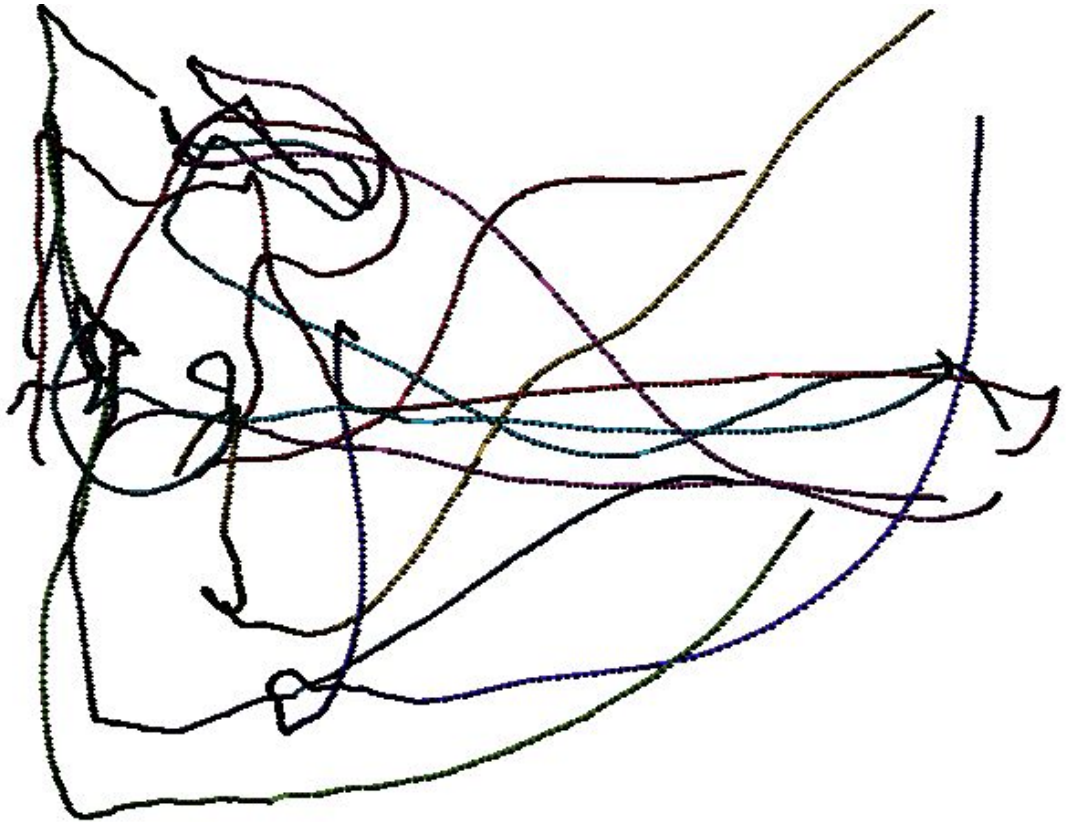




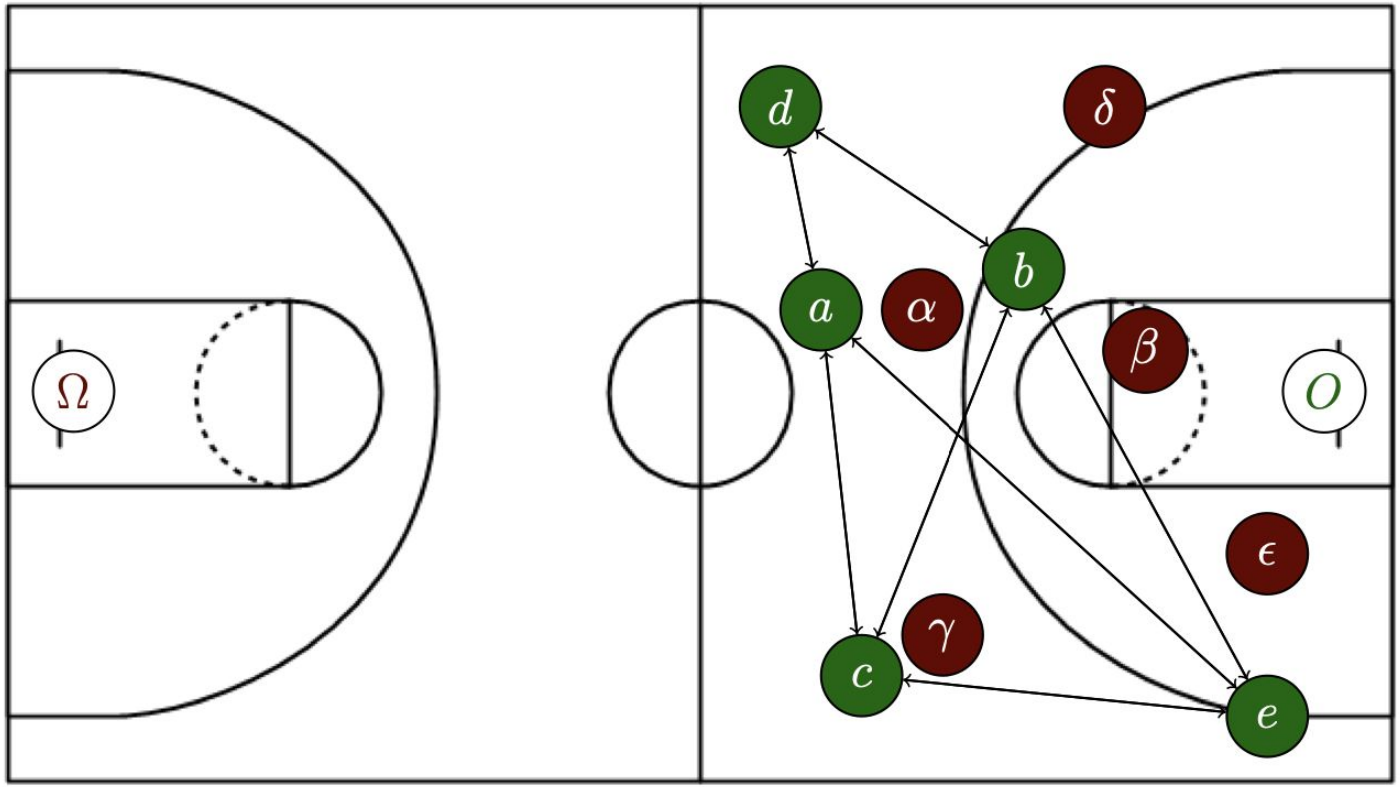
NASA: Satellites

III. Machine Learning

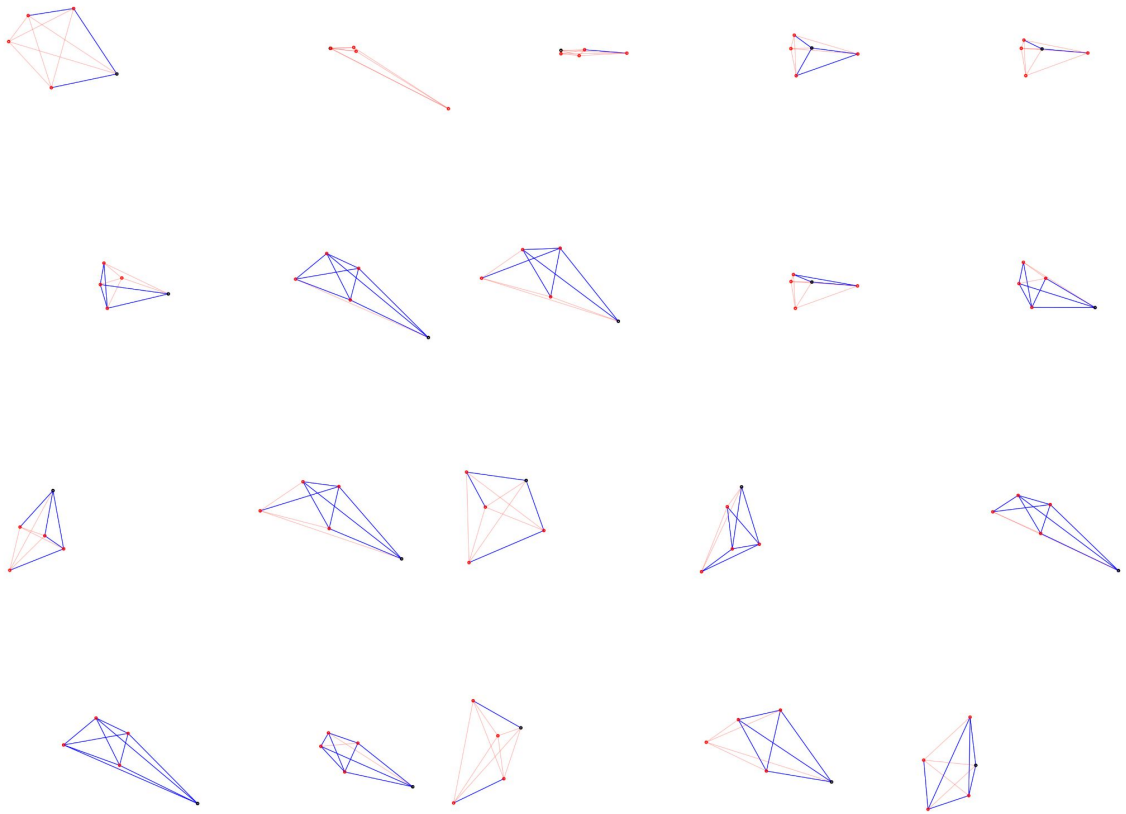
Duke v. UNC (booooo!)



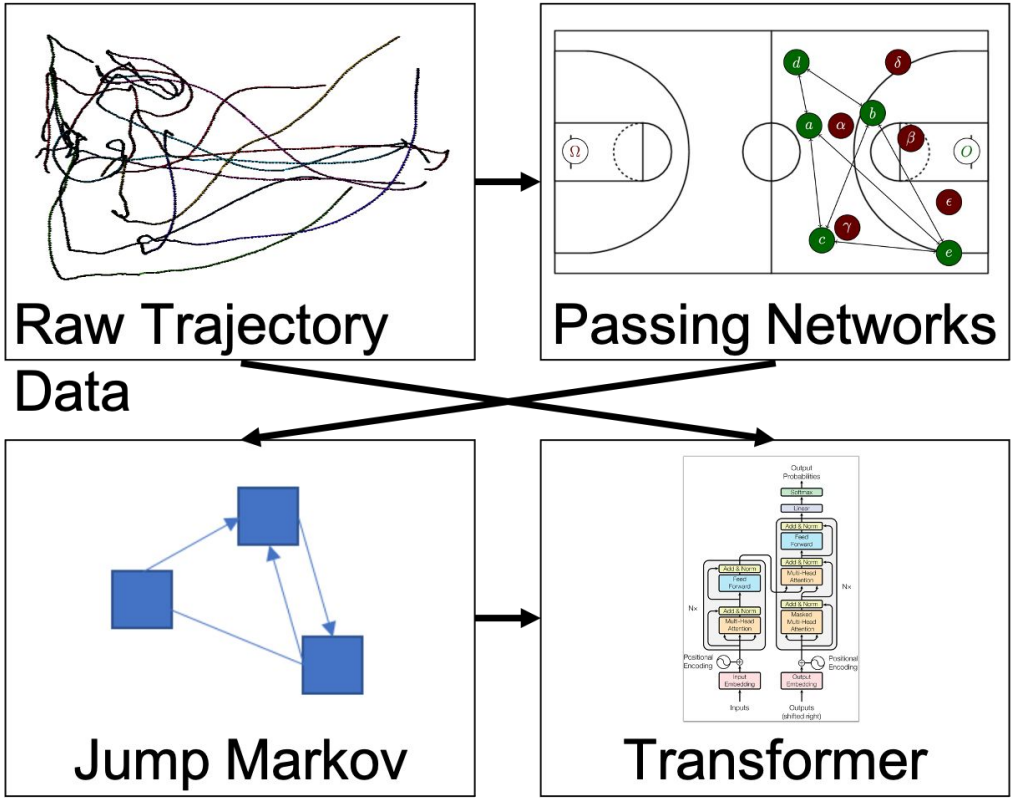
Raw Trajectory Data



Convert into Passing Network



Passing Graphy Library


Raw Trajectory Data
Passing Networks
Jump Markov
Transformer

Pipeline

66%

reduction in loss against benchmark
(40–10 trajectory prediction task)

Q&A